4) $\mathcal{A}$ & $\mathcal{B}$ - σ-field.

$\tilde{P}(A) := \sup\{P(B) : B \subset A\} = P(\tilde{B})$ $\left(\tilde{B} \in \mathcal{B},\ \tilde{B} \subset A \quad \tilde{B} = \cup B_n,\ P(B_n) > \sup - \frac{1}{n},\ B_n \subset A\right)$

$\tilde{P}(A \cap B) := P(\tilde{B} \cap B)$

$\tilde{P}(A^c \cap B) := P(\tilde{B}^c \cap B) = P(B) - P(\tilde{B} \cap B)$

$C_i \in \sigma(\mathcal{B}, A)$ - disjoint $C_i = A \cap B_i \cup A^c \cap B_i'$.

$C_i$ - disjoint ⇒ $\tilde{B} \cap B_i$ - disjoint ($A \cap B_i$)

so $\tilde{P}(A \cap (\cup B_i)) = P(\tilde{B} \cap (\cup B_i)) = \sum \tilde{P}(A \cap \cup B_i)$

$\tilde{B}^c \cap B_i'$ - does not have to be disjoint, but

$(B_i' \cap B_j' \cap \tilde{B}^c) \subset B_i' \cap B_j' \cap (A \setminus \tilde{B})$

$B_i' \cap B_j' \cap \tilde{B}^c \subset A \setminus \tilde{B}$ ⇒

$P(B_i' \cap B_j' \cap \tilde{B}^c) = 0$! (otherwise, $P(\tilde{B} \cup (B_i' \cap B_j' \cap \tilde{B}^c)) > \tilde{P}(\tilde{B})$)

so we can use problem 6 to conclude everything.

$\tilde{P}(A^c \cap \cup B_i) = P(\tilde{B}^c \cap (\cup B_i)) = \sum P(\tilde{B}^c \cap B_i) = \sum \tilde{P}(A^c \cap B_i)$

(sets $(\tilde{B}^c \cap B_i)$ - almost disjoint).

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6) Inclusion - exclusion

$P(\cup A_i) = \sum P(A_i) - \sum P(A_i \cap A_k) + \sum P(A \cap \ldots)$ (0, 0)

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10) $P(A \cap B) = \max(P(A), P(B))$.

$A \cap B \subset A$
$\subset B$

$P(A \setminus B) = P(A) - P(A \cap B) = 0$

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13) $\sum P(B_k^c) < 1$ $\quad P$

$\sum P(B_k) \leq n$

$\sum P(B_k) > n-1$

$P(B_k^c) = 1 - P(B_k)$

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21) $\mathbb{N}$ $\quad \{A : A \text{ or } A^c \text{ is finite}\}$

$$\mu(A) = \begin{cases} 0, & A \text{ is finite} \\ 1, & A^c \text{ is finite} \end{cases}$$

$A_n = \{n, \dots\}$

$\mu(A_n) = 1 \quad \bigcap A_n = \emptyset$

$\mu(\bigcap A_n) \neq \lim \mu(A_n)$

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